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Application of Kalman Filter with Control Input to Stock Price Prediction

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Abstract

Prediction of stock prices is regarded as a difficult task due to its random-walk characteristics. Also, the problem of estimating unobserved latent variables from observed data often arises in stock price prediction. The concept of the Kalman filter serves as a key method used for estimating unobserved variables from observed variables because of its real-time dynamic tracking characteristics. In this work, we investigate the effect of incorporating control input into the Kalman Filter (KF) algorithm for stock price prediction. The KF algorithm was used to predict stock prices by estimating the stock's hidden state based on noisy observations taking into account the control input. The control input as used in this work is the effective interest rate. The control input was incorporated in the algorithm, to monitor its effect on the stock price prediction. The predicted estimates from the stock price data were evaluated using three evaluation metrics, the Relative Error (RE), Mean Square Error (MSE) and Root Mean Square Error (RMSE). The datasets used are stock prices from First Bank of Nigeria (FBN) and Microsoft Corporation (MSFT) for a one-year period, from August 2024 to September 2025. Numerical simulations were carried out using the MATLAB software. The results showed little increase in the stock prices as a result of the control input. The simulations for the evaluation metrics showed that the FBN stock price could be controlled with an error below 4% and MSFT with an error below 3%. Also, the metrics MSE_c *and* RMSE_c *were lower for the FBN and MSFT stock price data. This implies that incorporating control input into the KF algorithm improved*

estimation accuracy. The study showcases the potential of the Kalman filter as a valuable tool for stock price prediction with control input.

Keywords: Kalman filter, Stock price, Control Input, Prediction

1. Introduction

Over the years, stock price prediction has become an important undertaking in business and finance. The entire idea of predicting stock prices is to gain significant profit in investments. According to [5], accurate analysis of financial changes and stock price prediction provides operators and investors with reliable management of investment and decision making. The prediction of stock price using machine learning algorithm helps forecast the future value of a company's stock or other financial assets traded on an exchange [1]. It enables investors to make informed decision about buying and selling of stocks.

Kalman filter was discovered in the early 1960s by an American researcher, Rudolf E. Kalman. He introduced the concept of state variables and state space of dynamical systems. This led to the development of the concept of the Kalman filter [4]. Initially, it was used to monitor the position and the speed of an orbiting vehicle, however this method is now applied to non-physical systems like economic systems [9]. Kalman filter is a classic state estimation technique used in application areas such as signal processing and autonomous control of vehicles. It is constructed as a mean square error minimizer that estimates state variables of a discrete linear dynamic system, minimizing the estimation error covariance [2]. It is also seen as an optimal estimator used in various applications such as smoothing noisy data and providing estimates of parameters of interest [8].

In stock price prediction the Kalman filter is used to forecast future prices based on historical data and real-time measurements. The algorithm predicts stock prices by first estimating the stocks hidden state based on noisy observations then predicts the next state and finally updates the estimate with new measurements. The Kalman filter algorithm also takes into account external influences, which is the control input that could affect the dynamics of the system. A control input is a vector of exogenous inputs that affects the system's state or behavior. In the prediction of financial assets such as stock prices [7] utilized the Kalman-Filter to assess market efficiency in major stock markets. Their findings revealed significant portfolio returns in emerging markets such as Korean, Vietnamese, and Malaysian, as well as positive returns in developed markets like the UK, Europe, Japan, and Hong Kong. The results suggest that the Kalman-Filter-based price reversal indicator yielded promising results across various market types.

To monitor dynamic risk in the field of management, two market risk metrics were estimated (Value-at-Risk (VaR) and Expected shortfall (ES)) using the Kalman filter in a work by [6]. A variety of volatility estimates, including the Kalman filter's recursive approach, were used to measure the VaR and ES. Loss-forecast accuracy was also computed using standard methods and compared to the Kalman filter. Results show that the Kalman filter produced the most accurate loss forecast estimates in periods of both calm and volatile markets. It also provided the most accurate forecasts for future market risk losses compared with standard methods which resulted in a more accurate provision of regulatory market risk capital.

[10] established financing risk decision-making model based on the Kalman filter. The key variables affecting financing risk were extracted by principal component analysis.

The model was tested with 88 listed SMEs. The results showed that the average error between the predicted and the real values is 8.5% and the overall recognition accuracy of the model is 89.1%. Hence, they concluded that the risk decision-making model had high discriminant accuracy and can provide evidence to risk decision-making.

Although there are many presentations of Kalman filter in literature, they usually deal with particular systems like autonomous robots and applications in engineering [3], with fewer applications in the area of finance. In financial time series, most studies demonstrate the versatility and effectiveness of Kalman filters, particularly in risk management, portfolio optimization, and asset pricing, with less concentration on the effect of control inputs like interest rate on the systems. In this work the control input is applied and its effect on stock price prediction is investigated.

2. Methodology

2.1. The Kalman filter model

The Kalman filter is based on a discrete-time linear dynamic system perturbed by Gaussian noise. It consists of a state-transition equation and a measurement equation. The state-transition equation describes the evolution of the state from time k to time $k+1$:

$$\mathbf{x}_{k+1} = \mathbf{A}_k\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k. \tag{1}$$

Here, $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is the state vector at time k, $\mathbf{A}_k \in \mathbb{R}^{n_x \times n_x}$ is the state-transition matrix, $\mathbf{u}_k \in \mathbb{R}^{n_u}$ is the control-input vector, and $\mathbf{B}_k \in \mathbb{R}^{n_x \times n_u}$ is the control matrix. The process noise $\mathbf{w}_k$ is assumed to have a multivariate normal distribution with mean zero and covariance matrix $\mathbf{Q}_k$:

$$\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k). \tag{2}$$

The measurement (observation) equation relates the state vector to the observed data:

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k, \tag{3}$$

where $\mathbf{y}_k \in \mathbb{R}^{n_y}$ is the observation vector, $\mathbf{H}_k \in \mathbb{R}^{n_y \times n_x}$ is the measurement matrix, and $\mathbf{v}_k$ is the measurement noise. The measurement noise is assumed to have a multivariate normal distribution with mean zero and covariance matrix $\mathbf{R}_k$:

$$\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k). \tag{4}$$

2.2. The Kalman filter iteration process

The Kalman-filter iteration has two stages: the time update (prediction) and the measurement update (filtering). In the stock-price application, the effective interest rate is used as the control input. The model without a control input is obtained by setting $\mathbf{u}_k = \mathbf{0}$.

First stage: Time update (prediction). At the first stage, the predicted state estimate is calculated from the previous updated estimate. With control input, it is

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_k\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k\mathbf{u}_k. \tag{5}$$

For the model without control input, set $\mathbf{u}_k = \mathbf{0}$ in Equation (5). The predicted error-covariance matrix is

$$\mathbf{P}_{k|k-1} = \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathsf{T}} + \mathbf{Q}_k, \tag{6}$$

where $\hat{\mathbf{x}}_{0|0}$ and $\mathbf{P}_{0|0}$ are the initial state estimate and initial error-covariance matrix, respectively.

Second stage: Measurement update (filtering). The Kalman gain determines the weight assigned to the measurement innovation, which is the difference between the observed measurement and its predicted value. It is calculated as

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathsf{T}} \left(\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathsf{T}} + \mathbf{R}_k \right)^{-1}. \tag{7}$$

The updated state estimate and error-covariance matrix are then

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \left(\mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \right), \tag{8}$$

$$\mathbf{P}_{k|k} = \left(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k \right) \mathbf{P}_{k|k-1}. \tag{9}$$

The iteration process for the Kalman-filter algorithm is summarized in Figure 1.

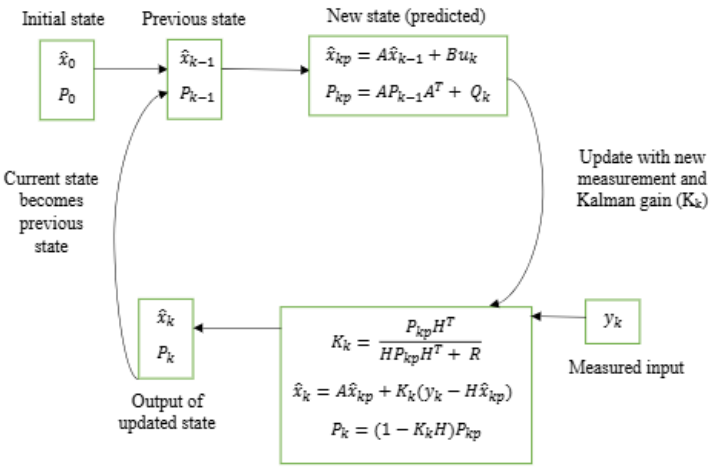


Figure 1: Kalman filter iteration flow chart

2.3. Evaluation metrics

In this work, three evaluation metrics are considered; relative error (RE), mean square error (MSE), and root mean square error (RMSE). These evaluation metrics will help explain the performance of the Kalman filter technique by evaluating the filtered estimates.

Relative Error (RE): The relative error provides a measure of error relative to the true value's size. It is the absolute difference between the estimated value $\hat{x}_i$ and actual value y_i, relative to the actual value, often expressed as a percentage.

$$\mathrm{RE}_i = \frac{|y_i - \hat{x}_i|}{|y_i|} \times 100\%. \tag{10}$$

Mean square Error (MSE): This metric measures the average squared difference between the actual value and estimated values (.

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{x}_i)^2. \tag{11}$$

Root Mean Squared Error (RMSE): This metric is the square root of the MSE and provides a measure of the average magnitude of the error.

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{x}_i)^2}. \tag{12}$$

where n is the number of measurements, y_i is the measurement value, and $\hat{x}_i$ is the corresponding filtered estimate (predicted value).

3. Result and Discussion

3.1. Numerical simulation of Kalman filter

In this work, filtering and prediction is carried out using the Kalman filter algorithm for two stock-price datasets: First Bank of Nigeria (FBN) stock price and Microsoft Corporation stock price (MSFT). The datasets were obtained from Yahoo Finance (https://ng.investing.com), a reliable and widely used source for financial data. We focus primarily on the closing stock price for each data set, which covers a one-year period from 1st August 2024 to 30th September 2025. A control input is incorporated into the Kalman filter algorithm to monitor its effect on the prediction of the stock prices. The control input as used in this work is the interest rate for the year 2025 for each of the stock price data. Numerical simulations for each of these data sets is implemented using the MATLAB software, showing the Kalman filter estimates without control input ($\hat{x}_k$) and the Kalman filter estimates with the control input ($\hat{x}_{kc}$). In order to monitor the effect of the control input on the stock prices, the relative error (RE), mean square error (MSE), and root mean square error (RMSE) are computed.

3.1.1 Numerical simulation for FBN stock price

FBN is one of the largest and most iconic banks in Nigeria, headquartered in Lagos. The effective interest rate for 2025 is 0.25%. (https://www.firstbank.com)

Using initial state estimate x0 = 27.76, measurement noise (standard deviation) R = 3.45, initial covariance estimate P = 1, initial process noise Q = 0 and interest rate of 0.25%, the filtered estimates are obtained and presented in Table 2 below.

Table 1: Filtered estimates for FBN stock price

k	y_k	$\hat{x}_k$	$\hat{x}_{kc}$	K_k
1	20.39	26.10	27.74	0.08
2	20.39	25.05	27.72	0.07
3	20.49	24.34	27.70	0.07
4	20.54	23.83	27.68	0.06
5	21.48	23.55	27.66	0.06
⋮	⋮	⋮	⋮	⋮
285	27.30	24.92	27.35	0.001
286	27.30	24.96	27.35	0.001
287	27.30	24.99	27.35	0.001
288	27.35	25.02	27.36	0.001
289	27.75	25.05	27.36	0.001

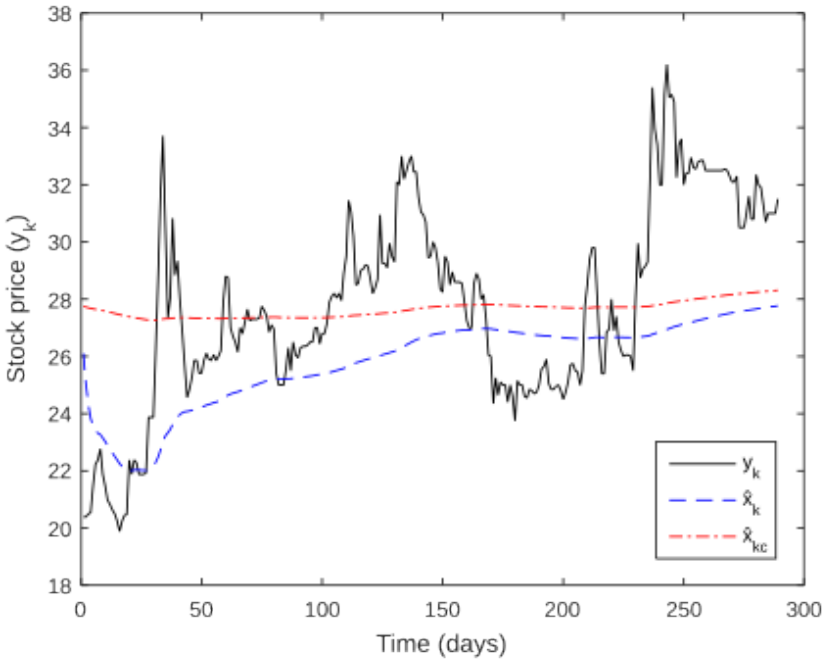


Figure 2: Filtered estimates for FBN stock price

From Figure 1, the FBN stock price started at \$21 and peaked at \$34 before 50 days. The filtered estimate $\hat{x}_k$ started with \$26 and remained below \$28. The figure also shows a significant effect of the control input from day 1 to day 125, however after day 150 its effect appears to reduce. ($\hat{x}_{kc}$) started below \$28 and remained below \$28.

3.1.2 Numerical simulation for MSFT stock price

MSFT is a multinational technology company headquartered in Redmond, Washington, USA. The effective interest rate as of September 2025 is 4.61%. (https://www.gurufocus.com/term/effective-interest-rate/MSFT/). Using initial state estimate x0 = 441.56, measurement noise (standard deviation) R = 44.72, initial covariance esti-

mate P = 1, initial process noise Q = 0 and interest rate = 4.61%, the filtered estimates are obtained.

Table 2: Filtered estimates for MSFT stock price

k	y_k	$\hat{x}_k$	$\hat{x}_{kc}$	K_k
1	420.79	441.11	441.54	0.08
2	412.49	440.49	441.49	0.07
3	389.17	439.42	441.38	0.07
4	400.00	438.61	441.30	0.06
5	408.84	438.01	441.24	0.06
⋮	⋮	⋮	⋮	⋮
285	510.62	440.72	450.10	0.00
286	508.30	440.92	450.23	0.00
287	510.06	441.13	450.37	0.00
288	511.50	441.34	450.51	0.00
289	513.24	441.56	450.65	0.00

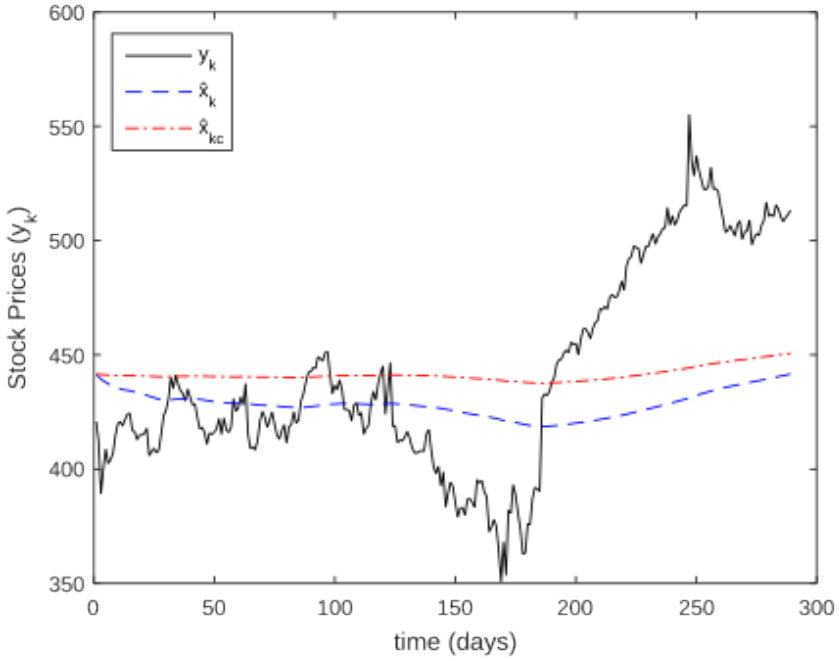


Figure 3: Filtered estimates for MSFT stock price

The MSFT stock price dropped to $350 after 150 days and peaked above $550 before day 250; however, the filtered estimate $\hat{x}_k$ remained below $450. The control input as seen in Figure 2, does not show a significant difference from the estimate without control. The filtered estimate with control ($\hat{x}_{kc}$) also remained below $450 after 250 days.

3.2. Evaluation of the filtered estimates

Evaluating the filtered estimates is critical to understanding the performance of the Kalman filter technique. In this section, the filtered estimates will be evaluated using the

Relative Error (RE), Mean Square Error (MSE) and (RMSE). RE is the relative error from the filtered estimates without the control input and RE_c is the relative error with the control input, Likewise, MSE and RMSE are the mean square and root mean square errors from the filtered estimates without control. MSE_c and $RMSE_c$ are the mean square and root mean square errors from the filtered estimates with control. These errors are calculated for all the datasets. The summary of the results are given in Table 3 below. The difference between the relative errors are obtained to check how they vary from each other. These differences d_i are given by:

$$d_i = RE_i - RE_{c,i}, \qquad i = 1, 2, 3. \tag{13}$$

where,

RE is the relative error calculated for the filtered estimate without control and RE_c the relative error calculated for the filtered estimate with control

3.2.1 Evaluation of the filtered estimates for FBN stock price

The values of RE and RE_c are presented in Figure 3 below. The relative errors (RE) and RE_c of FBN stock price remain below 4% as shown in Figure 3. Hence the FBN stock price can be controlled with an error below 4%. The Mean square errors are MSE = 11.6267 and MSE_c =11.0093 and the Root mean square errors are RMSE =3.41 and $RMSE_c$ = 3.32. The metrics MSE_c and $RMSE_c$ are lower than MSE and RMSE, indicating that the control input improved the estimation accuracy. The difference d_2 in Figure 4 also indicates low variation between the metrics since most of the values fall below zero.

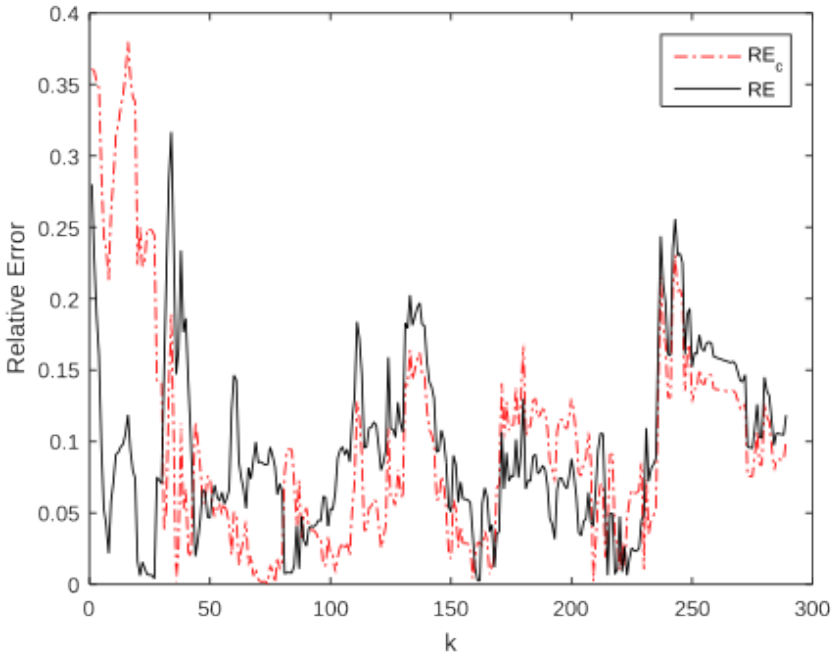


Figure 4: Relative errors for FBN stock price

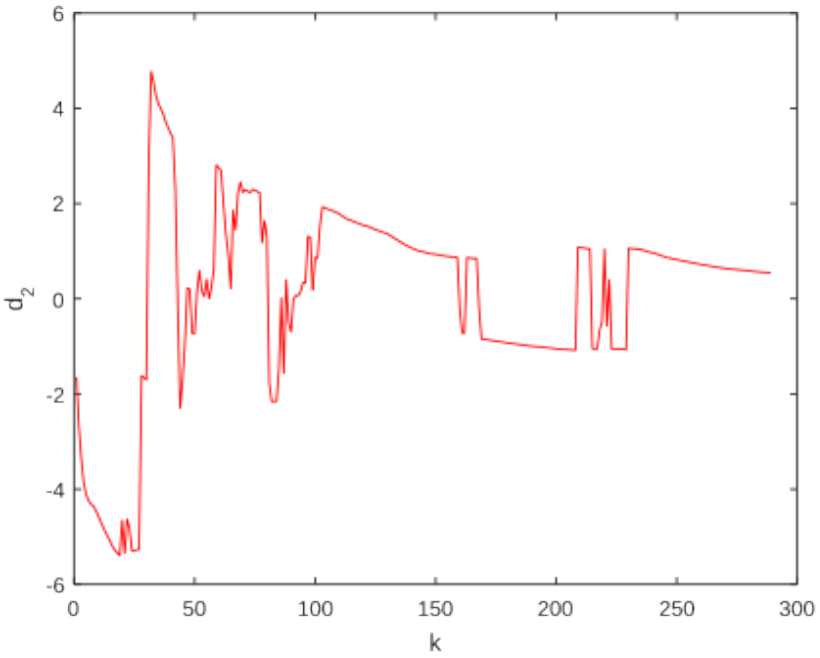


Figure 5: Plot showing the difference d_2

3.2.2 Evaluation of the filtered estimates for MSFT stock price

From Figure 5, the maximum relative error for MSFT stock price is RE = 2.3% and RE_c = 2.5% as shown in Figure 5. Hence the MSFT stock price can be controlled with an error below 3%. The mean square errors are MSE = 0.00199 and MSE_c =0.00182 and the Root mean square errors are RMSE =0.045 and $RMSE_c$ = 0.043. This also shows that, incorporating the control input reduced the estimation error. The distribution of the values of d_3 in Figure 6 shows a low variation between the metrics RE_c and RE.

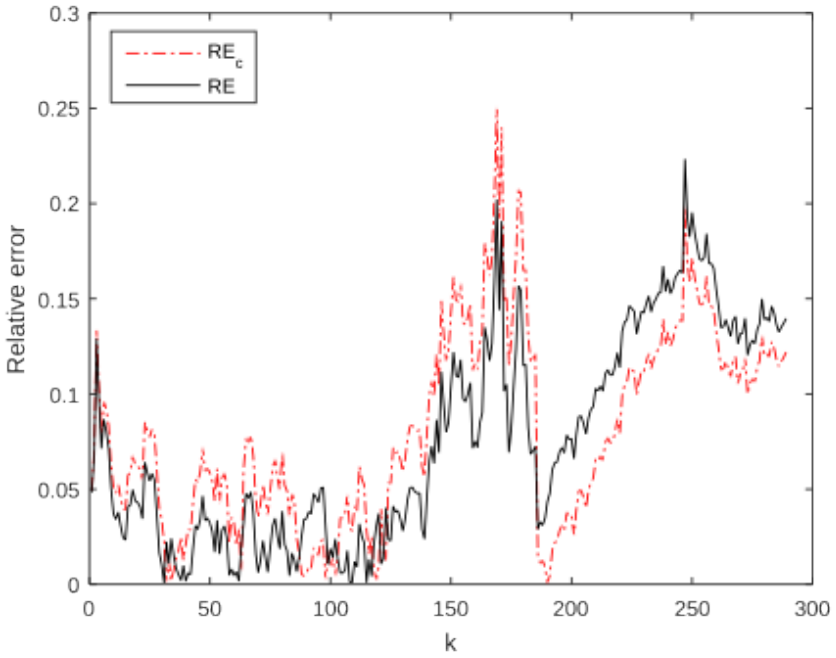


Figure 6: Relative errors for MSFT stock price

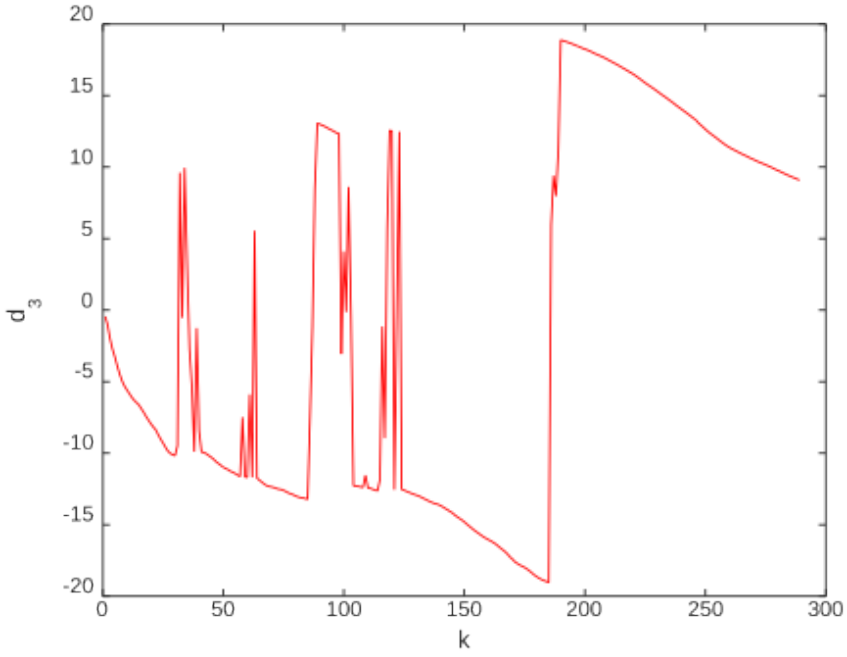


Figure 7: Plot showing the difference d_3

Table 3: Summary of the results for MSE and RMSE

Dataset	MSE	MSE_c	RMSE	$RMSE_c$
FBN	11.6267	11.0093	3.41	3.32
MSFT	0.00199	0.00182	0.045	0.043

4. Conclusion

This study applied the Kalman filter technique to stock-price prediction with control input. The Kalman filter works by recursively predicting the state of the system and updating the prediction based on new measurements. First Bank of Nigeria (FBN) and Microsoft Corporation (MSFT) stock price data were considered for a one-year period, from 1 August 2024 to 30 September 2025. The evaluation metrics RE, MSE and RMSE were used to evaluate the predicted estimates with control ($\hat{x}_{kc}$) and without control ($\hat{x}_k$) to check the performance of the Kalman Filter with control input. The results showed that the FBN stock price could be controlled with an error below 4% and MSFT with an error below 3%. The difference in the error distributions with control input concentrated more around zero, indicating reduced estimation errors. Results also demonstrated that the Kalman filter with control input provided accurate estimates, with a significant reduction in MSE_c and RMSE_c compared to the MSE and RMSE for the FBN and MSFT stock prices. This implies that incorporating control input could improve estimation accuracy. The study showcases the potential of the Kalman filter as a valuable tool for stock price prediction with control input.

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