DOI: 10.5281/zenodo.21307850 | Volume: 1 | Issue: 1 | ISSN: 3141-643X

Ktrend - International Journal of Computational Mathematics and Scientific Computing

STOCHASTIC ANALYSIS OF EUROPEAN PUT OPTION ON SHARE PRICE CHANGES PRICES

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ABSTRACT

In this paper, the Black-Scholes model of put option were investigated on the share prices of Fidelity, Access and Merged Banks; which paved way to obtain put option close form prices. The table results were presented on disparities of put option prices at specified time frame and the effect of the relevant parameters were discussed. Also, the put option prices with two different maturity dates shows that investor or the banks are more flexible to adjust positions based on changing market conditions. A larger difference between two put option prices with different expiration dates generally indicates a greater degree of volatility in the price of the underlying security to be more volatile, and will generally price the option with longer expiration date higher reflecting the increased potentials for the option to be in the money at expirations for Fidelity , Access banks and Merged banks. In comparing the share prices of the banks; it was discovered that merged bank has the highest mean and standard deviation of share prices which inform investors or bank management that the stock will perform well and likely grow in value. Finally, analysis of share prices were conducted using minimum variance criteria of each independent banks under-study where proposition were stated and proved to show levels of share price changes.

Keywords: Put option, Share price, Stock market, European option and stochastic analysis.

1. INTRODUCTION

The stock market is one of the most vital components of a free-market economy. It is among the best options to various companies for expansion or set up a new business venture. The stock market performance and operation has been widely recognized as a significantly viable investment field in financial markets. Investments can be done in stock, bond, mutual funds etc.

However, the nature of stock prices has been unstable, seasonal, time-dependent and highly volatile and therefore unpredictable. This is mostly due to uncertainties that arise from natural calamities, global trends, socio-political policies which may have unprecedented impact on the demand and supply of stocks [1]. Because of this, investors now have to go beyond studying the company's history, performance and development prospects of such fundamentals, but also be familiar with the variety of technical analysis in order to win a huge return on investment and become a successful investor. Stock trend analysis plays an important role in practical stock trading . One of the best option is to choose stocks by fundamental analysis and then confirm when to buy and sell stocks by technical analysis. Hence the need for stochastic analysis of Black-Scholes of option pricing is the appropriate model. Conversely, an option is a tool whose worth is derived from the principal asset which is otherwise known as financial derivative. This type of derivative does not have anything in common with mathematical meaning of derivative. In other words, an option on underlying asset is a business between parties who come together to agree on either buying or selling an underlying asset at a determined strike price in the future for a fixed price, [2]. The Black-Scholes formula is a mathematical model to calculate the price of put and call options. Since put and call options are distinct, there are two formulas, which account for each option. Call options give the option holder the right to buy the underlying stock for an agreed upon price anytime between today and upon expiration. Traders that believe the underlying stock will go up over time buy these call options in the hopes of making money. On the flip-side, put options give the option holder the right to sell the underlying stock for an agreed upon price anytime between today and upon expiration.

Conversely, many researchers have applied Black-Scholes model in different methods; for instance, [2] examined the impact of Crank-Nicolson Finite Difference approach in Valuating of Options. so ,[3] stipulated the rate of the option lies on the underlying asset, which is frequently a stock, commodity, currency or an index. In another dimension [4] established a new technique of assessing pricing effects on the premise to reduce pricing bias. [5] used the tempered fractional derivative to price a European-double-knock-out barrier option. [6] examined Black-Scholes model analysis and violated the assumptions of BS which says that volatility is constant.

However, [7] showed that the Black-Scholes model has been a major advance in finance over a period of time. [8] posited that since the Black–Scholes Option pricing model has long been in use for valuation of equity options to find the price of stocks. [9] proposed a high accurate method based on non-standard Runge–Kutta , modified weighted essentially non-oscillatory. In the work of [10] the Laguerre neural network was proposed as a novel numerical algorithm with three layers of neurons for solving BS equations. Recently [13] studied the perception of European Call option , the explicit price on the variations of maturity days is found accordingly.The application of Black-Scholes cannot be over emphasized that is, why so many authors has extensively written on it such as follows: [11],[12],[15-19] etc.

More so, the aim of this paper is to develop empirical approach of finding stochastic analysis of European put option on share price changes. It has been known that investors are really affected in their primary decisions due to expected returns. This motivated the authors of this paper to develop a vital and good approach that can stand as tool in making vital decisions.

It is obvious that [13] has considered analysis of Black-Scholes model of option pricing with time varying parameters on assumption that two call option prices do not come from a common distribution through Komogorov Sminorf. The advantage of current paper over [13] is that a larger difference between two put option prices with different expiration dates generally indicates a greater degree of volatility in the price of the underlying security to be more volatile, and will generally price the option with longer expiration date higher reflecting the increased potentials for the option to be in the money at expirations for Fidelity , Access banks and Merged banks, the analysis of share prices were conducted using minimum variance criteria of each independent banks under-study where proposition were stated and proved to show levels of share price changes. Our novel idea compliments previous efforts and extends the frontier of knowledge in this dynamic area of mathematical finance.

The plan of this paper is set as follows: Section 2.1 is Mathematical framework of Black-Scholes model, Section 3.1 Results and Discussion and conclusion are seen in Section 4.1.

2. Mathematical Framework of Black-Scholes Model

This model is commonly used in financial modeling. The Black-Scholes model is made up of seven assumptions: The asset price has characteristics of a Brownian motion with μ and σ as constants, the transaction costs or taxes are not allowed, the entire securities are absolutely divisible, dividend is not permitted during the period of the derivatives, unacceptable of riskless arbitrage opportunities, the security trading is constant, the option is exercised at the time of expiry for both call and put options.

In mathematical finance, an arbitrage arguments show that any derivative $V(S,t)$ written on v must satisfy the partial differential equation of the form of option pricing; hence we have the following:

:

$$\frac{\partial V(S,t)}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V(S,t)}{\partial S^2} + rS\frac{\partial V(S,t)}{\partial S} - rV(S,t) = 0. \tag{1}$$

$$V(S,t) \to \infty \quad \text{as } S \to \infty \text{ on } [0,T]. \tag{2}$$

$$V(S,t) \to 0 \quad \text{as } S \to 0 \text{ on } [0,T]. \tag{3}$$

And final time condition given by :

$$V(S_T,T) = (S_T - K)^+ = f(S_T) \text{ on } [0,\infty]. \tag{4}$$

Where r represents interest rate, σ represents volatility of the underlying assets and τ represents time of maturity.

With boundary conditions: Equation (3) is the value of asset which is worthless when the stock price is zero, The details of the above option model can be expressly found in the following [13] and [21] etc.

To eliminate the price process in (1) slightly gives the Black-Scholes analytic formula for the prices of European call option is given as follows

$$\left.\begin{aligned} C &= SN(d_1) - Ke^{rt}N(d_2), \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \\ d_2 &= d_1 - \sigma\sqrt{\tau} \end{aligned}\right\} \tag{5}$$

where C is Price of a call option, S is price of underlying asset, K is the strike price,r is the riskless rate , τ is time to maturity, σ^2 is variance of underlying asset,σ is standard deviation of the underlying asset (generally referred to as volatility), and N is the cumulative normal distribution. Similarly Black-Scholes analytic formula for the prices of European Put option is given as follows

$$\left.\begin{aligned} P &= SN(d_1) - Ke^{rt}N(d_2), \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \\ d_2 &= d_1 - \sigma\sqrt{\tau} \end{aligned}\right\} \tag{6}$$

Where P is the price of put option and the meaning of other parameters remain the same as in

2.1. The Initial Share Prices of: Access, Fidelity and When Merged

This represents share fair values of the option at the time it's issued for each independent banks. This fair value is determined by a number of factors, including the current share price, the expected volatility of share price and the time of expiration. The fair value is used to calculate the premium that the option buyer pays to the option seller. In other words, the butal share price of the put option indicates the value of the option at the time it is issued and gets the basis for calculating its price over times. It worthy to note that, the merged bank was discovered by [14] in their study on share price movements.

2.2. Analysis of Share Price Changes

Let $X_{it}, \rangle = 1, \ldots \tau$ be the share prices of Fidelity F(t)), Access A(t) and Merged M(t) banks. Also let an N x N share price data metrics be associated with each independent banks be $A_i t$. Considering each metrix from where further statistics are derived. The best selection of share prices was made based on minimum variance criterion, that is Min (Var), $(V_i\ t,\ i = 1,\ 2,\ 3)$; [4] and [20].

Proposition 1. Suppose Fidelity (F(t)), Access (A(t)) and Merged (M(t)) banks have some money in a risk free current account at time t. assuming their investment grows according to a continuously compounded interest rate r all through the trading days, then their value increase by a factor $t^n e^{rt}$. Where n represents the number of share prices for each bank. More so, an amount at a time t worth the minimum variances:$\gamma_1,\ \gamma_2$and γ_3 of Fidelity, Access and Merged banks respectively. If the trading of shares or adjustment of portfolios is considered due to rate of returns of investment and by recovering past trading of shares prices, the dynamics, follows thus:

i. $\int \gamma_1 t^n e^{rt} dt\ for\ \ F(t) = \gamma_1 \int t^n e^{rt} dt\ for\ \ F(t)$

ii. $\int \gamma_2 t^n e^{rt} dt\ for\ \ A(t) = \gamma_2 \int t^n e^{rt} dt\ for\ \ A(t)$

iii. $\int \gamma_3 t^n e^{rt} dt \ for \ \ M(t) = \gamma_3 \int t^n e^{rt} dt \ for \ \ M(t).$

Proof

To show different levels of share price changes using the dynamics in (i), (ii), and (iii).

Appling the Nedu's formula for integration by part, we obtain the solution to the integrals.

$$\int_{k=0}^{n} t^n e^{rt} dt = \sum_{k=0}^{n} (-1)^k P^{(k)}(t) \int_{k=0}^{(k+1)} f(t) dt$$

Where $P^{(k)}$ is the kth derivative of P(t) and $\int^{(k+1)} f(t)dt$ is the (k+1)st integral of f(t). Then we have

$$\begin{aligned}\int t^n e^{rt}\, dt = P(t) \int f(t)\, dt - P^{(')}(t) \int^{('')} f(t)\, dt \\ + P^{('')}(t) \int^{(''')} f(t)\, dt - P^{(''')}(t) \int^{('v)} f(t)\, dt \\ + \ldots + (-1)^n P^{(k)}(t) \int^{(k+1)} f(t)\, dt.\end{aligned}$$

$$\begin{aligned}\int t^n e^{rt}\, dt &= \frac{t^n e^{rt}}{r} - n\frac{t^{n-1} e^{rt}}{r^2} + n(n-1)\frac{t^{n-2} e^{rt}}{r^3} \\ &\quad - n(n-1)(n-2)\frac{t^{n-3} e^{rt}}{r^4} + \ldots \\ &\quad + (-1)^n n! \frac{t^{n-n} e^{rt}}{r^{n+1}} \\ &= \sum_{k=0}^{n} (-1)^k \frac{k! t^{n-k} e^{rt}}{r^{k+1}} + C.\end{aligned}$$

Thus the level of share price changes for Fidelity bank is shown to be

$$F(t) = \gamma_1 \sum_{k=0}^{n} (-1)^k \frac{k! t^{n-k} e^{rt}}{r^{k+1}} + C_1. \tag{7}$$

That of Access bank is

$$A(t) = \gamma_2 \sum_{k=0}^{n} (-1)^k \frac{k! t^{n-k} e^{rt}}{r^{k+1}} + C_2. \tag{8}$$

That of Merged bank is

$$M(t) = \gamma_3 \sum_{k=0}^{n} (-1)^k \frac{k! t^{n-k} e^{rt}}{r^{k+1}} + C_3. \tag{9}$$

3. RESULTS AND DISCUSION

In this Section we present the computational results for the problem formulated in Section 3.1. The table results are implemented in Matlab programming language.

Table 1: The value of put option prices for Access Bank, Plc and variations of maturity days: r = 0.03, σ = 0.25, k = 450, t = 1

Initial share price S_o	Put option prices when time t=6	Put option price when time t=12	Differences in put option prices ΔP
410	402.4696	407.5614	5.0918
80	71.1419	72.2696	1.1277
126	118.1743	120.8531	2.6788
79	70.0954	71.1908	1.0954
98	89.7471	91.5046	1.7575
92	83.5869	85.1300	1.5431
127	119.1833	121.8911	2.7078
91	82.5567	84.0641	1.5074
378	370.4692	375.5130	5.0438

As can be seen in Table 1, when the maturity time of a put option on the share price of Access Bank increase, the value of the put option also increase. This is because as the maturity time increases, the put option become less sensitive to changes in the underlying share price. Theoretically, this is because time value of the put option, which is the difference between the current value of the option and its intrinsic value, increases as maturity time increase, the put option holder has more time to potentially benefit from a decrease in the share price.

The volume of the initial share price of access bank can indeed affect the value of a put option on the share. This is because, in order to determine the value of a put option, one must first calculate in fair value of the share. This value is determined by taking into account the current share price, as well as the expected future value of the share.

Table 2 The value of put option prices for Fidelity Bank, Plc and variations of maturity days: r = 0.03, σ = 0.25, k = 450, t = 1

Initial share price S_o	Put option prices when time t=6	Put option prices when time t =12	Differences in put option prices ΔP
415	407.4697	412.5678	5.0981
62	51.9957	52.6824	0.7254
138	130.2642	133.2727	3.0085
61	50.9091	51.5845	0.6754
121	113.1249	115.6538	2.5289
81	72.1867	73.3473	1.1606

Initial share price S_o	Put option prices when time t=6	Put option prices when time t =12	Differences in put option prices ΔP
139	131.2702	134.3043	3.0341
80	71.1419	72.2696	1.1277
384	376.4693	381.5232	5.0539

It can be seen be in Table 2 that has helped put option values than Fidelity bank It could suggest that investors perceive access bank to be more risky than fidelity bank, and are willing to pay more for the right to sell its shares at a set price in the future. Alternatively. It could mean that the current share price of access bank is higher than fidelity bank, or that of access bank's volatility is higher. In either case, this could have implications for the company's future performance and the value of its shares. It could be with dipping deeper with into the data to understand.

Table 3: The value of put option prices for Merged Bank, Plc and variations of maturity days. r = 0.03, σ **= 0.25, k = 700**

Initial share price S_o	Put option prices when time t=6	Put option prices when time t=12	Differences in put option prices ΔP
825	817.4701	822.7180	5.2479
142	134.2872	137.3962	3.109
264	256.4616	261.1338	4.6722
140	132.2761	135.3354	3.0593
219	211.4464	215.7844	4.338
173	165.3936	169.1393	3.7457
266	258.4620	263.1453	4.6833
171	163.3894	167.1008	3.7114
762	754.4701	759.7128	5.2427

In Table 3, the two banks merged there shares which result to several financed implications. For instance, the merged entity would likely have a larger market capitalization, or value, then either of the individual banks. This could make it easier for the merged bank to raise capital such as through a stock offering. Additionally, the merged bank might be able to take advantage of economies of scale and synergies, such as sharing back office functions, which could reduce costs and improve efficiency. Finally, the merger could lead to increased computation in the banking sector which could benefit consumers.

Generally in Tables 1,2 and 3 shows the put option prices with two different maturity dates shows that investor or the banks are more flexible to adjust positions based on changing market conditions. The difference between two put option prices of different expiration dates can provide insight into market's expectations for the underlying security's future price movements. A larger difference between two put option prices with different expiration dates generally indicates a greater degree of volatility in the price of the underlying security to be more volatile, and will generally price the option with longer expiration date higher reflecting the increased potentials for the option to be in the money at expirations for Fidelity , Access banks and Merged banks.

Table 4: The Statistics of Fidelity, Access and Merged Bank Share Prices

Fidelity share price			Mean	STD	Variance	Skewness	Min Variance
415	62	138	205	175.75	30889	1.4	
61	121	81	87.6607	30.55	933.33	0.936	933.33
139	80	284	167.6667	105.00	11022,34	1.16	
Access share price			**Mean**	**STD**	**Variance**	**Skewness**	**Min Variance**
410	80	126	205.33	178.70	31947.13	1.64	
79	98	92	89.67	9.71	94.30	-1.02	94.30
127	91	378	198.67	156.34	24449.41	1.64	
Merged			**Mean**	**STD**	**Variance**	**Skewness**	**Min Variance**
825	142	264	410.33	364.26	132682.30	1.52	
140	219	173	177.33	39,67	1574.72	0.50	39.67
127	171	762	353.33	354.60	125742.33	1.71	

Min Var (933.33, 94.30, 39.67)

More so, maximum variance of the share price indicates that a security's returns are more spread out and less predictable. this can mean more risk, but also potentially higher returns if the security performs well. Minimum variance in other hand indicates that a security's returns are more spread out and less predictable.

Predictable and stable. this can mean less risk, but potentially lower returns as well.

Ultimately, investors in Fidelity and Access banks must weigh the potential risks and rewards of each security to determine which investments align with their financial goals and risk tolerances.

The interpretations of Table 4.5 are as follows: The share price data was used to form matrices of each independent bank where all the statistics are derived row-wise. (See column 1 to 4 of Fidelity, Access and Merged banks Share Prices).

The big mean share price could be seen as a positive sign for investors, as it could indicate that the stock is performing well and is likely to continue to grow in value. They could be a good investment opportunity for investors who are looking for stocks with strong growth potential. A small mean share price, on the other hand, could be seen as a sign of weakness or under-performance (see column 2).

A big share price with high standard deviation and variance could indicate that the stock is highly volatile and risky, but with the potential for higher returns. A small share price with high standard deviation and variance could indicate that the stock is under-performing and may not be a good investment (see column 3 & 4 respectively).

As can be seen in table 4 column 5 shows the skewness of the share price data. Skewness provides some valuable insights into the behavior of share prices. When a share price has positive skewness, it means that there are more data points on the positive side of the distribution, indicating that the share price has a tendency to rise more frequently than it falls. This is generally seen as a good sign for investors, as it suggests that the share price has a strong upward trend.

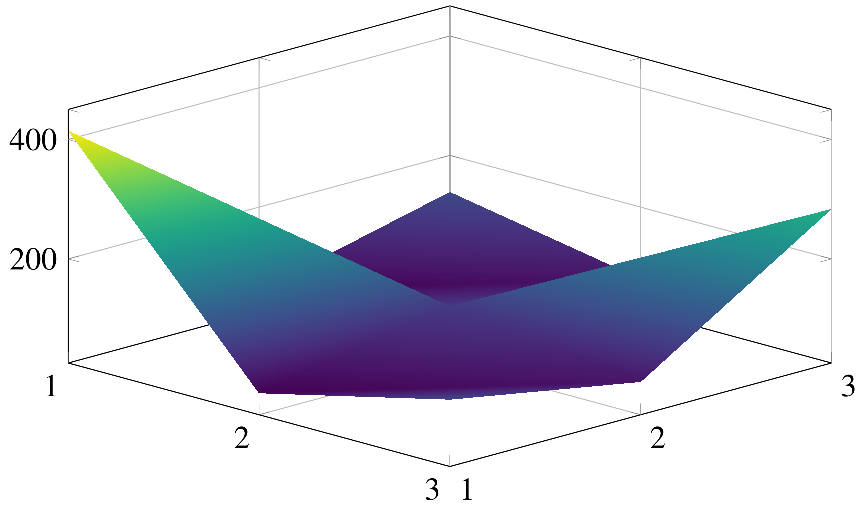

Figure 1: Surface view of Fidelity Share prices in matrix form

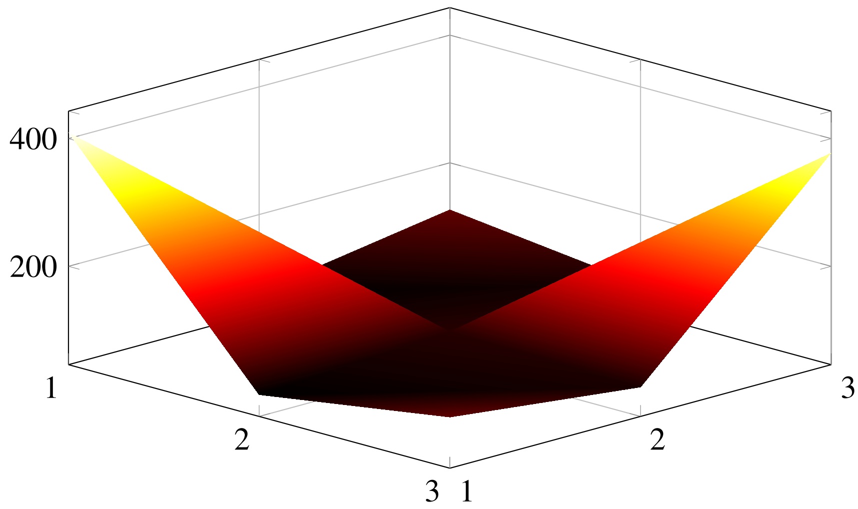

Figure 2: Surface view of Access bank Share prices in matrix form

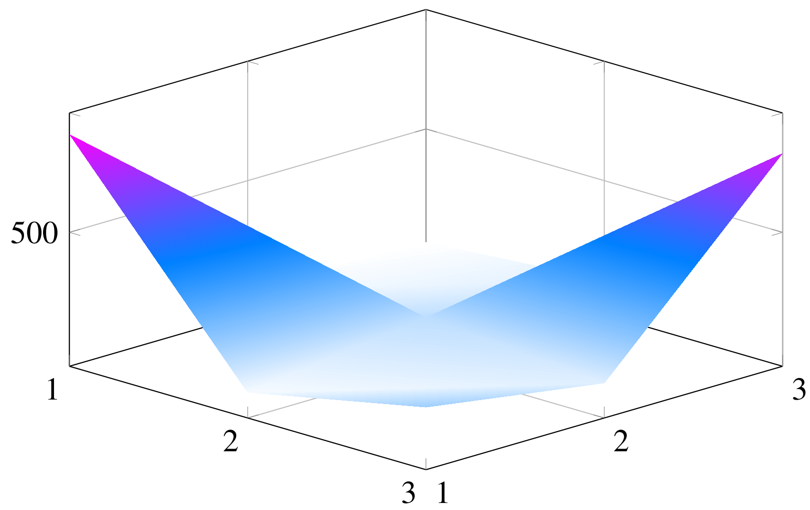

Figure 3: Surface view of merged bank share prices in matrix form

Figures 1-3 are descriptive illustrations of surface view results of Fidelity, Access and Merged banks share prices. It describes levels of changes of existence on the value of share prices. The shapes of the surface view shows unsatisfactory amplitudes show-casing its uncertainty due to stochastic formations. The trajectories seen on the figures is the price history of stock market which is been determined by volatility.

3.1. Application of Section 2.4: Determining future levels of share price changes

Hence, the minimum variances of the three banks will be subjected to analysis in other to ascertain different levels of shares prices, we state the following.

Proposition 1: Suppose Fidelity (F(t)), Access (A(t)) and Merged (M(t)) banks have some money in a risk free current account at time t. assuming their investment grows according to a continuously compounded interest rate r=0.3 all through the trading days, then their value increase by a factor $t^n e^{rt}$.

Where n represents the number of share prices for each bank. More so, an amount at a time worth the minimum variances: 30.55, 9.71 and 39.67. If the trading of shares or adjustment of portfolios is considered due to rate of returns of investment and by recovering past trading of shares prices, the dynamics, follows thus:

iv. $\int 30.55t^9 e^{0.3t} dt \; for \;\; F(t)$

v. $\int 9.71t^9 e^{0.3t} dt \; for \;\; A(t)$

vi. $\int 39.67t^9 e^{0.3t} dt \; for \;\; M(t)$

Proof.

To show different levels of share price changes using the dynamics in (i), (ii), and (iii)

From (i): $\int (30.55)(\, t^9 e^{0.3t})dt$

$$30.55 \int t^9 e^{0.3t} dt$$

Using Nedu's method of integration by parts.

$$= 30.55 \left\{ \frac{t^9 e^{0.3t}}{0.3} - \frac{9t^8 e^{0.3t}}{0.09} + \frac{72t^7 e^{0.3t}}{0.027} - \frac{504t^6 e^{0.3t}}{0.0081} + \frac{3024t^5 e^{0.3t}}{0.00243} - \frac{15120t^4 e^{0.3t}}{0.000729} + \frac{60480t^3 e^{0.3t}}{0.0002187} - \frac{181440t^2 e^{0.3t}}{0.00006561} + \frac{362880te^{0.3t}}{0.000019683} - \frac{362880e^{0.3t}}{0.0000059049} + k_1 \right\}.$$

$$= 30.55e^{0.3t}\left\{\frac{t^9}{0.3} - \frac{9t^8}{0.09} + \frac{72t^7}{0.027} - \frac{504t^6}{0.0081} + \frac{504t^7}{0.0081} + \frac{3024t^5}{0.00243} - \frac{15120t^4}{0.000729} + \frac{60480t^3}{0.0002187} - \frac{181440t^2}{0.00006561} + \frac{362880t}{0.000019883} - \frac{362880}{0.0000059049}\right\} + k_1.$$

Where K_1 is the constant of integration which is assumed to be share price volatility of fidelity.

ALSO

ii) $\int 9.71t^9e^{0.3t}dt$

- $9.71 \int t^9e^{0.3t}dt$

$$= 9.71\left\{\frac{t^9e^{0.3t}}{0.3} - \frac{9t^8e^{0.3t}}{0.09} + \frac{726t^7e^{0.3t}}{0.027} - \frac{504t^6e^{0.3t}}{0.0081} + \frac{3024t^5e^{0.3t}}{0.00243} - \frac{15129t^4e^{0.3t}}{0.0000729} + \frac{60480t^3e^{0.3t}}{0.0002187} - \frac{181440t^2e^{0.3t}}{0.00006561} + \frac{362880te^{0.3t}}{0.000019663} - \frac{362880e^{0.3t}}{0.0000059049} + k_2\right\}.$$

$$= 9.71e^{0.3t}\left\{\frac{t^9}{0.3} - \frac{9t^8}{0.09} + \frac{72t^7}{0.027} - \frac{504t^6}{0.0081} + \frac{3024t^5}{0.00243} - \frac{15120t^4}{0.000729} + \frac{60480t^3}{0.0002187} - \frac{181440t^2}{0.00006561} + \frac{362880t}{0.000019883} - \frac{362880}{0.0060059049}\right\} + K_2.$$

Where K_2 is the constant of integration which is assumed to be the volatility of Access Bank.

Similarly

iii) $\int 39.67t^9e^{0.3t}$

- $39.67 \int t^9e^{0.3t}dt$

$$= 39.67\left\{\frac{t^9e^{0.3t}}{0.3} - \frac{9t^8e^{0.3t}}{0.09} + \frac{72t^7e^{0.3t}}{0.027} - \frac{504t^6e^{0.3t}}{0.0081} + \frac{3024t^5e^{0.3t}}{0.00243} - \frac{15120t^4e^{0.3t}}{0.000729} + \frac{60480t^3e^{0.3t}}{0.0002187} - \frac{181440t^2e^{0.3t}}{0.00006561} + \frac{362880te^{0.3t}}{0.000019683} - \frac{362880e^{0.3t}}{0.0000059049}\right\} + K_3.$$

$$= 7.2794\, e^{0.3t}$$

$$\left\{\frac{t^9}{0.3} - \frac{9t^8}{0.09} + \frac{72t^7}{0.027} - \frac{504t^6}{0.0081} + \frac{3024t^5}{0.00243} - \frac{15120t^4}{0.000729} + \frac{60480t^3}{0.0002187} - \frac{181440t^2}{0.00006561} + \frac{362880t}{0.00001983}0.000019883 - \frac{362880}{0.0000059049}\right\} + k_3.$$

Where K_3 is the constant of integration which is assumed to be share price volatility of future Merged bank.

Proposition 1: shows the levels of future changes in the share prices which provides important information to investors about the health of the banks and its prospect for future growth. The future changes might indicate: Upward price movement. When share prices are rising, it often indicates that the market is bullish and expects it to perform well in future. It is also a signal for investors to buy as there might be potential profit from the growth. Downward price movement. When share prices are falling, it often indicates that there is concern about the bank's performance or future prospects.

4. Conclusion

This paper considered analytical solution of Black-Scholes on share price of Fidelity, Access and Merged Banks. The study shows the inequalities on the specified time frame is obtained effectively which shows that increase in specific time frame increases the value of put option for the three banks under study. Also, the put option prices with two different maturity dates shows that investor or the banks are more flexible to adjust positions based on changing market conditions. A larger difference between two put option prices with different expiration dates generally indicates a greater degree of volatility in the price of the underlying security to be more volatile, and will generally price the option with longer expiration date higher reflecting the increased potentials for the option to be in the money at expirations for Fidelity , Access banks and Merged banks. The merged Banks has the highest mean and standard value which is informative to the bank management. To this end, the analysis of share prices were conducted using minimum variance criteria where proposition were developed and proved to determine some levels of share prices charges.

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Conflict of Interest

The authors declare no conflict of interest.

Funding Statement

This research received no external funding.